

Timber harvest scheduling in a fuzzy decision environment

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Linear programming is a widely used tool for timber harvest scheduling in North America. However, some potential problems related to infeasible harvest schedules, overly optimistic objective function values, and the need to strictly satisfy all constraints included in deterministic model formulations have been raised. This paper describes a fuzzy approach for explicitly recognizing the imprecise nature of the harvest flow constraints usually included in harvest scheduling models. The objective function and selected constraints are viewed as soft, and satisfactory solutions are derived and discussed for several scenarios. An illustrative sample problem is presented to demonstrate the methodology, and a comparison with solutions derived from a traditional linear programming model is presented.

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La programmation linéaire est un outil largement utilisé pour la planification des programmes de coupe dans les forêts d'Amérique du Nord. Cependant, les problèmes potentiels reliés à des programmes de coupe non réalisables et à des fonctions objectives ayant des valeurs trop optimistes ainsi que la nécessité de satisfaire sans exception toutes les contraintes inhérentes à la formulation d'un modèle déterministe ont été soulevés. Cette publication décrit une approche nébuleuse pour tenir compte de façon explicite de la nature imprécise des contraintes de flux de coupe normalement incluses dans les modèles de programme de coupe forestière. La fonction objective et les contraintes choisies sont considérées comme étant flexibles et les solutions satisfaisantes sont dérivées et analysées pour plusieurs scénarios de coupe. Un cas-type est présenté à titre d'exemple pour démontrer la méthodologie et une comparaison est faite avec les solutions obtenues en utilisant un modèle de programmation linéaire traditionnel.

[Traduit par la rédaction]

Introduction

The use of optimization models, particularly linear programming (LP), in timber harvest scheduling has become commonplace in North America. This acceptance is primarily because of the capability of LP to optimize any linear objective and at the same time meet any set of linear constraints that may restrict the attainment of the objective. This flexibility, coupled with ease of both computation and model interpretation, has stimulated usage of this modeling tool.

While LP is a suitable planning tool for timber harvest scheduling, some criticisms have been raised concerning its use. One of the most frequent criticisms is that LP treats all model parameters as nonstochastic measurements that are known with certainty (Bare and Field 1987). Many attempts have been made to incorporate uncertainty into LP models. Past studies include stochastic LP using chance constraints (Thompson and Haynes 1971), LP with penalty cost minimization (Marshall 1988), and LP with a multistage recourse formulation (Hoganson and Rose 1987). These approaches generally follow classical concepts of stochastic programming and probability theory. Most, however, have been used sparingly in actual practice.

More recently, Gassmann (1989) applied stochastic programming using the Dantzig-Wolfe decomposition principle to determine an optimal harvest schedule in the presence of risk of forest fires. It was found that stochastic models

produced more conservative results than deterministic counterparts. Reed and Errico (1986) applied LP to a similar problem and reported optimum harvest volumes to decrease as the per-annum fire probabilities increased.

Hof *et al.* (1988) were among the first to examine the problem of using LP in timber harvest scheduling models when timber yield coefficients in the constraint set were treated as random variables. They concluded that a solution generated without recognizing the randomness of the timber yield coefficients nearly always is infeasible. Pickens and Dress (1988) also examined this phenomenon and arrived at similar conclusions. Further, they found that the objective function value obtained when the constraint set contained random timber yield coefficients was an optimistically biased estimate of the true expected value. Hobbs and Hepenstal (1989) reached similar conclusions for the case where random estimation errors in objective function coefficients resulted in an upward bias in the optimal objective function value, assuming that errors in the constraint set were relatively unimportant.

This paper describes how fuzzy mathematical programming can be used to cope with uncertainty in timber harvest scheduling models. Unlike the papers cited above, it is assumed throughout that uncertainties can be adequately modeled as fuzzy sets. Thus, timber yield coefficients are treated as deterministic, but (i) strict satisfaction of constraint limits is relaxed and (ii) attainment of goal aspiration levels is sought,

but not required. Solutions derived using a fuzzy formulation are contrasted with results obtained from a traditional LP-based approach.

Fuzzy approach to model uncertainty

As an alternative to probability-based models, a fuzzy approach has been proposed for modeling uncertainty. This approach was developed by Zadeh (1965) primarily in response to the need to solve problems with fuzzy data (e.g., measurement inaccuracies or vagueness of available information). Although the literature does not endorse a single unique definition of fuzziness, there are some qualitative differences in the kind of imprecision and uncertainty for which the fuzzy approach was developed, and for which probabilistic or stochastic approaches may not be appropriate. As stated by Mendoza and Sprouse (1989), "the concept has generally been associated with complexity, vagueness, ambiguity, and imprecision." This further implies that model coefficients, parameters, or functional relationships may be fuzzy and, hence, not known with complete certainty.

Besides probabilistic methods, sensitivity analysis has also been proposed to handle imprecision and uncertainty. This type of analysis examines the best and worst cases based on ranges of values for the parameters. Parametric analysis is another possible approach for investigating the impacts of uncertainty on model behavior. However, this method also becomes unwieldy when many parameters are examined simultaneously.

Fuzzy optimization offers an alternative framework for modeling uncertainty and imprecision. Since the seminal work of Zadeh (1965), the literature has expanded rapidly. Readers are referred to the following references for details: Zimmermann (1985, 1987), Dubois and Prade (1980), and Negoita (1979). For forestry applications, Hof *et al.* (1986), Mendoza and Sprouse (1989), and Pickens and Hof (1991) are among the first published works dealing with fuzzy optimization in forest management.

Fuzzy linear programming

Fuzziness can be modeled in several ways depending upon the nature of imprecision, the context in which uncertainty occurs, and how it is accommodated in the problem. For instance, in a mathematical programming setting, fuzziness can be restricted to the constraints, the objective function, or both; and fuzziness may be manifested as fuzzy numbers (i.e., coefficients in the objective function or constraints) or as fuzzy sets (i.e., the objective function or constraints). In this paper, fuzziness is assumed to appear only in the objective function and the timber harvest flow constraints. Other constraints of the LP model are treated as crisp constraints.

The problem addressed represents a situation where the decision maker (DM) tolerates some degree of violation in the accomplishment of the timber harvest flow constraints. In USDA Forest Service national forest planning, this represents the case where departures from strict nondeclining flow over time are tolerated. In other applications this might represent violation of area regulation constraints, even-flow timber harvest flow constraints, or any form of harvest flow constraint wherein a periodic harvest flow must be within a given percentage of a previous period's harvest level. In these situations, the fuzzy constraints are to be satisfied as well as possible, but they do not have to be satisfied in a strict sense. Thus, these fuzzy constraints also can be referred to as soft

or flexible. If the membership function (see discussion below) invoked in the fuzzy formulation is linear, then we speak of a fuzzy LP (FLP) model.

Before formally defining the FLP model, we first restate the classical LP problem as follows:

$$[1] \quad \max. \quad c^T x$$

subject to

$$[2] \quad Ax \leq b$$

where [1] is the objective function and [2] is the set of linear constraints. Another constraint set, $x \geq 0$, is also understood.

In timber harvest scheduling models, [1] corresponds to a forest management objective (often maximization of net present value (NPV) or timber harvest volume) and [2] corresponds to management constraints such as acreage, budget, and harvest volume flow. For example, c could represent a NPV per acre and x the number of acres assigned to a specific management prescription. Under traditional LP formulations, the right-hand sides (b), representing resource limitations, are assumed to be fixed, and any solution (x) must strictly satisfy this absolute bound. Obviously, this assumption is very rigid, especially for complex problems such as those encountered in timber harvest scheduling. If the decision environment is completely known, the technical input-output coefficients in matrix A can be accurately determined and b also can be precisely specified. Under these assumptions, the formulation in [1] and [2] is a valid model of the decision problem. However, this is usually not the case in timber harvest scheduling because timber yield coefficients may be imprecisely estimated and strict adherence to constraint limitations may not be required. Given this situation, it is impractical to impose strict satisfaction of [2].

Most timber harvest schedules are revised after one or two planning periods have elapsed. Thus, in actuality, an iterative process is used to set timber harvest levels over time. If not linked to the harvest level immediately prior to recalculation, the new harvest schedule often results in a harvest volume that is inconsistent with the previous schedule. That is, harvest volumes may decline in actuality when (in theory) they were previously constrained to be nondeclining. This effect is discussed by McQuillan (1986) and Pickens *et al.* (1990) and is partial justification for treating the timber harvest flow constraints as soft and not as fixed requirements.

It is also possible to model the decision environment by treating the coefficients of A as random variables. This is not done in this paper because the added model complexity is not justified by the decision environment. Further, it is more appropriate to treat this type of problem using probability-based methods because timber yield coefficients are better viewed as a stochastic, and not a fuzzy, process. Following Negoita (1979), the decision environment is changed such that b_i is no longer a point estimate but instead becomes an interval. That is, constraint set [2] is changed to

$$[3] \quad \sum a_{ij} x_j \leq \sim b_i \quad \text{for all fuzzy } i$$

where $\leq \sim$ implies a fuzzified version of constraint [2]. Note that the a_{ij} and the objective function are treated as crisp. Operationally, b_i in [3] is represented as an interval (e.g., $[b_i, (b_i + t_i)]$), where t_i is the maximum amount of constraint tolerance permitted.

Intuitively, [3] implies that the DM tolerates a certain amount of violation in [2], but the constraints should be satisfied to the extent possible. This desire to jointly satisfy all flexible constraints may be characterized by the membership function, $m_i(x)$, as follows:

$$[4] \quad m_i(x) = \begin{cases} 1 & \text{if } a_i x \leq b_i \\ [(b_i + t_i) - a_i x]/t_i & \text{if } b_i < a_i x \leq b_i + t_i \\ 0 & \text{if } a_i x > b_i + t_i \end{cases}$$

An intuitive explanation of the membership function, $m_i(x)$, is as follows: the DM is very satisfied (i.e., the degree of satisfaction is equal to 1) if a solution x satisfies the constraint fully; the DM is less satisfied with a solution that meets the constraint only partially (i.e., within the tolerable amount of violation); and the DM is completely unsatisfied (i.e., the degree of satisfaction is equal to 0) if a solution x violates the constraint plus the maximum tolerable amount. Zimmermann (1978) proposed a linear membership function as in [4] for the situation where the degree of satisfaction decreases in a linear fashion (i.e., $0 \leq m_i(x) \leq 1$) for any solution with increasing constraint violation. The formal FLP model with flexible constraints can be described as

$$[5a] \quad \max. c^T x$$

subject to

$$Ax \leq \sim b$$

Note that [5a] implies that all constraints are fuzzy. This is unnecessary and can be relaxed by adding an additional set of crisp linear constraints as shown below:

$$[5b] \quad \max. c^T x$$

subject to

$$Ax \leq \sim b$$

$$Dx \leq d$$

The objective function ($c^T x$) is assumed to be a strict maximization, and a_{ij} and d_{ij} are crisp coefficients. Negoita (1979) refers to [5a] and [5b] as a flexible mathematical programming problem.

Problem [5] was originally presented by H.-J. Zimmermann¹ and Tanaka *et al.* (1974), and a number of procedures have been proposed to solve this formulation. For instance, given a linear membership function, Chanas (1983) has shown that [5a] and [5b] reduce to a parametric LP problem that can be solved to determine the entire fuzzy decision set.² However, instead of determining the entire fuzzy decision set, it may be more desirable to generate a single crisp solution to the fuzzy decision problem. Werners (1984, as cited by Zimmermann 1987) demonstrated that the following formulation generates one crisp solution to the fuzzy model shown in [5a] and [5b]:

$$[6] \quad \max. k$$

subject to

$$k(Z_0 - Z_1) - c^T x \leq -Z_1$$

$$kt + Ax \leq b + t$$

$$Dx \leq d$$

$$0 \leq k \leq 1$$

where Z_1 and Z_0 are the objective function values calculated by specifying the right-hand sides as b and $b + t$, respectively; t is the allowable relaxation (or tolerance limit) from each constraint, and k is an indicator of membership to the fuzzy set (representing constraints or objectives) that we wish to maximize (Zimmermann 1987). This formulation leads to one crisp solution, but it is not necessarily the best solution. Other solutions may be superior if nonlinear membership functions are used or if different values for t are selected.

If the objective function, as well as some constraints, is treated as fuzzy, an alternate formulation is required. For this situation we assume that we wish to find x such that

$$c^T x \geq \sim z$$

$$Ax \leq \sim b$$

$$Dx \leq d$$

Assuming linear membership functions as shown in [4] and now treating z as an aspiration level for the objective function, Zimmermann (1987) shows that an equivalent crisp formulation is

$$[7] \quad \max. k$$

subject to

$$kt + Bx \leq g + t$$

$$Dx \leq d$$

$$0 \leq k \leq 1$$

In [7], B contains both c and A , and g contains both z and b . Thus, [7] is fully symmetric with respect to the objective function and constraints. If present, multiple goals are assumed to be of equal importance in this formulation and are represented in B and g .

Ensuing sections of the paper utilize the model described by [6] to devise a harvest schedule that jointly seeks to maximize NPV while simultaneously achieving a nondeclining harvest flow to the extent possible. This model utilizes the MAXMIN operator to maximize k , the degree of satisfaction. In the first constraint of [6], Z_0 is the value of the objective function when t is set at its maximum tolerable limit, and Z_1 is the value of the objective when t is set equal to 0. The second constraint embodies the maximum tolerance in nondeclining yield acceptable to the DM. That is, while the DM prefers to have strict nondeclining harvest flow, a decline of t Mbf (1 Mbf = 1000 board ft; 1 board ft = 2.360 dm³) from one decade to the next is acceptable.

An examination of [6] reveals that the fuzzy nondeclining yield constraints are treated symmetrically with the objective of maximizing NPV. This approach (confluence of goals and constraints; Bellman and Zadeh 1970) generates one deterministic, or crisp, solution instead of the entire fuzzy decision set. Further, k jointly represents the degree of satisfaction in

¹H.-J. Zimmermann. 1974. Optimization in fuzzy environments. Paper presented at the Operations Research Society of America - The Institute of Management Sciences Meeting, San Juan, Puerto Rico.

²A fuzzy decision set consists of a number of solutions representing combinations of maximum $c^T x$ and satisfaction levels for the fuzzy constraints in eq. 5b.

terms of both the level of achievement for the objective and the satisfaction of the fuzzy constraints. Intuitively, maximizing the degree of satisfaction, k , implies that the DM is most satisfied when k approaches 1 and least satisfied when k approaches 0. Yet, it is highly unlikely for k to equal 1, since this occurs when yields decline at their maximum tolerated level. This is unlikely to occur since the second constraint in [6] seeks nondeclining yields.

Taking a different viewpoint, one also can interpret the maximum allowable violation of the nondeclining flow constraint to represent the expected uncertainty of the unknown parameter b . That is, b can be expected to have values within the interval $(b, b + t)$. In the context of the timber harvest scheduling sample problem, this interval is $(0, t)$ and may be interpreted as follows: while nondeclining flow is desirable, it is recognized that due to projection uncertainty, harvest volume for the next decade cannot be completely or accurately known. The degree of uncertainty is represented by the width of the interval.

Sample problem

To illustrate the potential usefulness and limitations of the fuzzy decision models described in the previous section, a timber harvest scheduling problem is presented and analyzed. Results derived from this model are contrasted with those derived using a traditional LP formulation. The sample problem is adapted from McQuillan (1986) and Pickens *et al.* (1990) and was selected because (i) it allows us to build on the work of Hof *et al.* (1986), (ii) it emulates the decision environment that occurs on many forest areas throughout the western United States, and (iii) it addresses a problem that timber managers face as they attempt to convert unmanaged forests into a more managed state.

The problem involves the maximization of NPV associated with timber harvesting over twelve 10-year planning periods subject to land area, nondeclining timber harvest flow, and long-run sustained yield capacity constraints. The last restriction limits periodic timber harvest volume to be less than or equal to an exogenously determined long-run harvest volume (17 857 Mbf/decade).

Using a model I formulation (Johnson and Scheurman 1977), all possible harvesting schemes are defined for the five analysis areas. Each area is 1000 acres (1 acre = 0.40 ha) in size and possesses identical site productivity, stocking, and timber quality characteristics. Existing stands yield 15 Mbf/acre when harvested, and regenerated stands yield 25 Mbf/acre when harvested at age 70 years (or later). The interest rate is 4%, the current product price is \$300/Mbf, the expected rise in prices is 1.5% per year for the first 50 years and constant thereafter, and harvesting costs for each analysis area are a constant \$170, \$290, \$410, \$530, and \$650/Mbf. All revenues and costs occur at the midpoint of each decade.

At the outset, 12 harvesting alternatives are defined for each analysis area. Each decision variable represents the number of acres assigned to an alternative within a particular area. Management alternatives allow for the harvest of existing stands with provision for a subsequent harvest in a later decade (if possible) in the planning horizon. In the initial formulation, there are 60 decision variables, 12 accounting variables representing the total harvest volume in each decade, 5 land area constraints, 11 nondeclining periodic

timber harvest flow constraints, 12 sustained yield capacity constraints, and 12 transfer rows that define total timber harvest volume.

Both McQuillan (1986) and Pickens *et al.* (1990) solve this problem using standard LP procedures. As they reported, the resultant timber harvest schedule for the initial planning iteration produces a nondeclining harvest flow of 10 000 Mbf/decade for the first 7 decades, followed by a harvest of 16 667 Mbf/decade for the remaining 5 decades in the planning horizon. Further, they demonstrate that if the optimal first period harvest schedule is adopted as shown, and a second iteration of planning is performed using the updated inventory that results if the first period solution is implemented, the resulting harvest volume in decade 2 drops to 9522 Mbf (a reduction of about 5%). And, successive planning iterations reveal a similar reduction in harvest volume over the first 4 decades. This phenomenon has been labeled "the declining even flow effect" (McQuillan 1986), and occurs in spite of the nondeclining timber harvest flow constraints that are mandated within a given planning iteration.

Two problems invite investigation: (i) How does a fuzzy approach to timber harvest scheduling compare with a LP approach within a given planning iteration? (ii) What role, if any, can fuzzy programming play in resolving the declining flow between planning iterations?

Comparison of FLP with LP

To demonstrate the potential usefulness of a fuzzy approach to help resolve the first of these questions, the sample problem described above is reexamined. As previously stated, the uncertainty or imprecision occurs in the constraint set involving the nondeclining flow requirements. All remaining constraints are treated as being crisp. Likewise, the objective function is assumed to be crisp and involves the maximization of NPV.

The FLP model described by [6] is used to generate a timber harvest schedule that maximizes the degree of satisfaction, k . For purposes of illustration, t is set equal to 500 Mbf for all 11 of the nondeclining flow constraints. In the context of the harvest scheduling problem, this results in the following constraints:

$$H_j - H_{j+1} + 500k \leq 500, \quad j = 1, 2, \dots, 11$$

where H_j represents the total harvest volume in decade j . To complete the definition of the FLP model shown in [6], values for Z_0 and Z_1 must be calculated. The latter value is that associated with the traditional nondeclining LP model, wherein t is set to 0. For the sample problem this produces a NPV of \$4 024 400. To obtain a value for Z_0 , a modified LP model must be solved. In the modified model, the nondeclining flow constraints are altered to read $H_j - H_{j+1} \leq 500$. This means that strict nondeclining flow can be violated so long as the reduction in total harvest volume from decade to decade does not exceed 500 Mbf. The results of this LP run produce a NPV of \$4 040 336.25, an increase of \$15 936.25 over the traditional nondeclining formulation.

The volume of timber scheduled for harvest under the two LP models just described is shown in Table 1. The larger NPV, associated with the Z_0 run, results in a larger 1st decade harvest, with a steady drop of 500 Mbf/decade until a low harvest volume of 9000 Mbf is reached in decade 4. The total timber harvest over the 12-decade planning horizon for the

TABLE 1. Comparison of harvest schedules using LP and FLP formulations (harvest volumes in Mbf)

Decade	LP with $H_j - H_{j+1} \leq 0$	LP with $H_j - H_{j+1} \leq 500$	FLP using eq. 6
1	10 000	10 500	10 250
2	10 000	10 000	10 000
3	10 000	9 500	9 750
4	10 000	9 000	9 500
5	10 000	10 714	10 375
6	10 000	10 286	10 125
7	10 000	9 786	9 875
8	16 667	17 500	17 083
9	16 667	17 000	16 833
10	16 667	16 500	16 583
11	16 667	16 000	16 333
12	16 667	17 857	17 291
NPV (\$)	4 024 400*	4 040 336.25†	4 032 378.75† ($k = 0.50$)

* Z_1 .† Z_0 .

TABLE 2. FLP solutions obtained using eq. 7 for differing NPV targets and tolerable deviations

Target NPV (\$)	Tolerable deviation from NPV target (\$)				
	5000	10 000	15 000	20 000	40 000
4 024 400					
k	0.76	0.62	0.52	0.44	0.29
NPV (\$)	4 028 208	4 030 551	4 032 137	4 033 282	4 035 818
4 032 379					
k	0.38	0.31	0.26	0.22	0.14
NPV (\$)	4 034 286	4 035 458	4 036 253	4 036 826	4 038 095

traditional nondeclining flow model is 153 335 Mbf, while the modified model results in a comparable volume of 154 643 Mbf. Both of these solutions are presented only for the first iteration of planning.

Given the values for Z_0 and Z_1 , the first constraint of [6] can be written as

$$-15\,936.25\,k + c^T x \geq 4\,024\,400$$

The results obtained using the FLP formulation shown in [6] are displayed in Table 1. The maximum value for k is found to be 0.50, indicating an equal trade-off between nondeclining harvest flow and NPV maximization. Consequently, the FLP formulation results in a maximum decline of only 250 Mbf between decades (one-half the tolerated amount) and results in achieving only one-half of the \$15 936.25 of NPV that is available to gain. These results are predictable, but the value of the minimum harvest volume over the 12-decade planning horizon is not. The minimum harvest volume (occurring in decade 4) has been increased to 9500 Mbf (up 500 Mbf from the traditional LP run).

Next, [7] is used with both the objective function as well as the nondeclining flow constraints treated as being fuzzy. Again, linear membership functions are assumed. Equation 7 represents a clear confluence of goals and constraints and illustrates how fuzzy goals and constraints can be combined. To complete the formulation of [7], an aspiration level for the NPV goal and a tolerable deviation from this target are

required. For purposes of comparison and illustration, a range of values is used to demonstrate the sensitivity of the resulting solution. The results of these calculations are shown in Table 2.

The two aspiration levels for NPV were selected to represent the minimum NPV associated with the strict nondeclining flow run and the value from the FLP using [6]. Tolerable deviations from these aspiration levels were picked to bracket the maximum \$15 936 difference between the two traditional LP runs shown in Table 1. Thus, with a \$4 024 400 NPV aspiration level and a tolerable deviation of \$15 000, the solution shown in Table 2 is comparable to the FLP solution of Table 1. The advantage of using [7] is that additional goal levels and tolerable levels of deviation can be examined and displayed. As expected, for a fixed NPV target, the results show a decreasing level of satisfaction over increasing levels of goal deviation. This is a consequence of increasing deviations from nondeclining flow being balanced against an increasing NPV. Also, as the NPV aspiration level increases, the overall measure of satisfaction decreases. All results shown in Table 2 continue to assume a maximum decline of 500 Mbf in the harvest flow constraints.

With regard to the second question (i.e., What role, if any, can fuzzy programming play in resolving the declining flow between planning iterations?), the results obtained above indicate that the usefulness of FLP appears quite limited. The principal problem here is one of inventory imbalance and

harvest yield as the forest structure is altered over time. This is further exacerbated by the sequential nature of the planning environment and the lack of linkages between successive planning iterations. The presence of declining flows under such circumstances indicates that adherence to strict non-declining flow within a planning iteration should be questioned, and perhaps abandoned. If so, FLP offers potential as a tool to allow the systematic exploration of satisfactory alternative solutions.

Conclusions

Two versions of a FLP timber harvest scheduling model are presented. The first assumes a crisp objective function combined with fuzzy nondeclining flow constraints, and the second assumes that the objective function is also fuzzy. Numerical results for both problems are shown and compared with previously published traditional LP formulations of the same problem. A brief discussion of the resulting solutions is presented.

Fuzzy methods, specifically FLP, appear to offer hope as viable planning tools for incorporating uncertainty into timber harvest scheduling problems. However, these methods are not substitutes for the more developed statistical or probabilistic methods, nor will they completely solve or shed additional light on all fuzzy problems. For example, the declining even flow effect cannot be eliminated or resolved simply by using FLP. But, FLP solutions provide additional insights not readily available from traditional LP solutions. Since FLP approaches are relatively easy to formulate and solve, they provide a useful tool for forest management analysts.

A critical component that requires additional study is the definition of the membership function (i.e., [4]). Perhaps a nonlinear function would better characterize harvest scheduling problems that involve nondeclining flow constraints. Such a formulation would enable a DM to generate a timber harvest schedule such that the degree of satisfaction would decrease dramatically as small violations from nondeclining yield arose, but then not decrease in proportion to larger violations. In light of the difficulty to fully characterize the uncertainty in timber harvest scheduling using other available techniques, FLP appears to be a good alternative planning tool that merits consideration.

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